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Hello, my name is Rowan Adeya, my supervisor for the research project was Professor Alexandra Olaya-Castro, and the title for my thesis was “Quantum Resources in Two-Level Systems coupled to quantum Harmonic oscillators”.

And what does that mean? Well, lets start by looking at what Two-Level Systems coupled to a quantum Harmonic oscillators are.

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Two-level system coupled to a quantum harmonic oscillator (or TLS-QHO for short) represents many physical systems, such as cavity QED, trapped ions, and dimers interacting with molecule vibrations.

The TLS part is defined as any system which has only two possible states, 0 for ground, and 1 for excited, and may be used to describe , for example, a qubit or two distinct energy states of an atom.

A QHO, on the other hand, is a system with evenly spaced energy levels and a quantised energy spectrum, and is described by the state vector psi QHO, where c n are our complex amplitudes. A QHO may describe a field, or vibrations in, say, a molecular system.

The coupled (or joint) system is then the tensor product of the two, and is written as capital psi.

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Now, lets move to Quantum resources: what are they? Quantum resources are fundamental properties of quantum systems. Two of the most important are entanglement and coherence.

Entanglement is a measure of how separable a joint system is, and is essential for quantum communication and cryptography and without it, procedures such as the BB84 protocol, or dense coding, wouldn’t be possible.

Coherence, on the other hand, reflects the superposition properties between subsystems, and is basis dependent. Under open system evolution (which we’ll talk about later), for example, these off-diagonals decay to the environment and the system is said to loss its coherence (decoherence). This is one of the key enemies of quantum computation, and maintaining coherence is vital for viable quantum computers.

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This decoherence leads us to one of the main complications of quantum resources in physical systems such as the ones described by a TLS-QHO. They are fragile, and in order to understand how to maintain them for longer periods, we must first understand how they come about in the first place.

Thus, our main goal is to find out how these Quantum resources manifest in TLS-QHO systems. Lets turn to the methods used in our research, before moving on to the results.

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First, lets choose two TLS-QHO models. We’ll be looking at the Jaynes cummings model, which models a single TLS coupled to one mode of our oscillator. Its one of the simplest models, and has been extensively studied, so was a good starting point for my research. We choose our parameters to mimic the low-temperature regime that describes, for example, cavity QED experiments.

We then look at a version of an Exciton Vibration model, which models a biological system where a dimer is described by one effective TLS, coupled to molecular vibrations which are described by a collective vibrational mode. As you can see, the Hamiltonian is more complex, and its less studied. The Hamiltonians differ primarily in the bare TLS Hamiltonian, and the interaction Hamiltonian. Our parameter choices model dimers responding to molecular vibrations in room-temperature experimental setups.

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Now, we need to evolve our models in time. There are two approaches we choose: closed evolution, and open evolution. Closed evolution ignores the environment, and the system is isolated, and is governed by the unitary operator here.

Open evolution includes the effects of the environment and is described by the Lindblad equation. The decay operators highlighted govern how the system loses information to the environment, and there are many decay channels we could consider.

We choose to look at spontaneous atomic emission, which affects the TLS, and thermal dissipation, which affects the oscillator.

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To evolve our models in time, we choose two initial conditions. Case 1 starts the TLS in an excited state, and our oscillator in its ground state. Case 2 looks at the TLS in a superposition, and the oscillator in its ground state. Both are in the computational basis and will remain so for ease of comparison.

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Now that we have our models, we know how we’ll evolve them, and we have defined our starting points, we can choose the measures of entanglement and. For entanglement, we choose the von Neumann entropy for closed evolution, since its only valid for pure states, and Negativity for open evolution, since its valid for mixed states.

For coherence, we use the relative entropy of coherence measure, as its valid for pure and mixed states of both evolutions.

So, we have our models, our starting points, and our evolution methods. Now, we can begin to examine how these resources manifest in our TLS-QHO models.

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Lets start by looking at the Jaynes cummings model under closed evolution. For our first case, we notice a key feature of the JCM: rabi oscillations. This refers to regular, sinusoidal oscillations in populations due to our interaction part of the Hamiltonian. We see that the system’s entanglement also oscillates, showing that rabi oscillations can maintain entanglement on average. We also see that, for the total system, coherence is present in the computational basis, and matches the period of entanglement. However, there is no coherence present in our subsystems.

Case 2 remedies this, and introduces coherence in both the subsystems, and actually boosts coherence overall. But, we see that our entanglement is lowered. This leads us to a key point: there is a trade-off between entanglement and coherence, and it is highly sensitive to our initial condition.

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We see this trade-off in action in our open system evolution. Whilst coherence is less in case 1 and entanglement is stronger, case 2 shows increased coherence at the expense of coherence. Interestingly, we see that entanglement decays twice as fast as coherence, which suggests that entanglement is perhaps the more fragile resource.

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Whilst we’ve seen regular, even patterns in the Jaynes cummings model, the exciton vibration model exhibits more interesting behaviour. We see three types of oscillations: high, medium and low frequency oscillations, which correspond to the bare frequencies of our system. Compared to the JCM, we get finer control of our system. When looking at entanglement, we see a similar picture for the EVM. Entanglement directly follows the natural oscillations of the system. We see the same for coherence. However, this time we have coherence in both cases for both subsystems, suggesting that systems described by the EVM can hold coherence in the subsystems. We again take note of the trade-off between entanglement and coherence, where in the second starting case, coherence is higher.

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Finally, for open evolution, we notice that there is a persistent residual coherence and entanglement under thermal dissipation, due to the revival mechanisms at higher temperatures.

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To conclude, we relook at our question: how do entanglement and coherence manifest in TLS-QHO systems. They appear due to oscillatory couplings between the two subsystems, and finer control of these resources can be achieved with more complex Hamiltonians. Entanglement is more fragile than coherence, shown by its faster decay times. There is also a trade-off between the two resources depending on the initial condition. Finally, higher temperature experiments may yield residual coherence and entanglement.

For our models, whilst the Jaynes Cummings model is a simple example which describes systems that can generate entanglement and coherence, the Exciton vibration model gives us more granular control of how entanglement and coherence manifest due to its finer oscillations.

Future simulations and experiments should look at how entanglement and coherence vary under different Exciton-Vibration system frequencies. Different initial states could also be analysed. Finally, we should look at different decay channels, as there are many more that we may consider.